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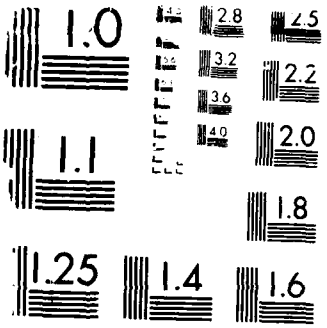

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This grant, issued under the University Research Instrumentation Program, provided Micro Vax II cmputers for experimental research in parallel methods in linear programming. The equipment permitted research in the solution of very large sparse linear programs and linear complementarity by successive overrelaxation (SOR) methods, especially the parallelization of SOR methods.

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Final Report

to

Air Force Office of Scientific Research

on

Instrumentation Grant No. AFOSR-86-0255:

"Primal-Dual Parallel Solution of Very Large Sparse Linear Programs"

Reporting Period:

July 30, 1986 - June 29, 1987

Principal Investigator:

Olvi L. Mangasarian
Computer Sciences Department
University of Wisconsin
Madison, WI 53706
(608) 262-6593, 262-1204
olvi@cs.wisc.edu

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1. Summary of Technical Results

Six technical reports were written in the period June 1, 1986-June 29, 1987 which made extensive use of the equipment provided by Grant AFOSR-86-0255. The titles [1]-[6] of these reports are listed in Section 2. The principal area of our research has been the solution of very large sparse linear programs and linear complementarity problems by successive overrelaxation (SOR) methods. Another important ingredient of our research has been the parallelization of our SOR methods as well as other classical methods such as the simplex method for linear programming and Lemke's method for the linear complementarity problem. A major contribution of our research has been the solution of one of the largest general linear programs ever attempted on a workstation (or in fact on a mainframe). We solved [3] a linear program with 500,000 variables, 125,000 constraints and 1,125,000 nonzero matrix elements, in less than 72 hours on one of the MicroVax II computers provided by this grant. None of the other state-of-the-art simplex packages could solve considerably smaller linear programs on the same machine. Another significant achievement of our research has been the parallelization of our SOR methods with speedup efficiencies sometimes exceeding 100%. The MicroVax II's were used to test simulations of the parallel SOR algorithms before their implementation on our multicomputers and multiprocessors. Details of our results are given in the enclosed six technical reports [1]-[6]. We give below summaries of each of these reports:

[1] A parallel successive overrelaxation (SOR) method is proposed for the solution of the fundamental symmetric linear complementarity problem. Convergence is established under a relaxation factor which approaches the classical value of 2 for a loosely coupled problem. The parallel SOR approach is then applied to solve the symmetric linear complementarity problem associated with the least norm solution of a linear program.

[2] A gradient projection successive overrelaxation (GP-SOR) algorithm is proposed for the solution of symmetric linear complementarity problems and linear programs. A key distinguishing feature of this algorithm is that when appropriately parallelized, the relaxation factor interval $(0, 2)$ is not reduced. In a previously proposed parallel SOR scheme, the substantially reduced relaxation interval mandated by the coupling terms of the problem often lead to slow convergence. The proposed parallel algorithm solves a general linear program by finding its least 2 norm solution. Efficiency of the algorithm is in the 50 to 100 percent range as demonstrated by computational results on the CRYSTAL token-ring multicomputer and the Sequent Balance 21000 multiprocessor.

[3] Serial and parallel successive overrelaxation (SOR) methods are proposed for the solution of the augmented Lagrangian formulation of the dual of a linear program. With the proposed serial version of the method we have solved linear programs with as many as 125,000 constraints and 500,000 variables in less than 72 hours on a MicroVax II. A parallel implementation of the method was carried out on a Sequent Balance 21000 multiprocessor with speedup efficiency of over 65% for problem sizes of up to 10,000 constraints, 40,000 variables and 1,400,000 nonzero matrix elements.

[4] We present a parallel asynchronous successive overrelaxation algorithm for the solution of symmetric linear complementarity problems and linear programs. A distinguishing feature of this algorithm is that processors need not communicate after each update of the solution vector and therefore processor idle time can be avoided. The proposed parallel algorithm is applied to finding least 2-norm solutions of linear programs. Improvement is observed over the synchronized version of the algorithm, the parallel gradient projection successive overrelaxation algorithm.

[5] We propose a two-stage successive overrelaxation (SOR) algorithm for solving the symmetric linear complementarity problem. After the first SOR preprocessing stage this algorithm concentrates on updating a certain prescribed subset of variables which is determined by exploiting the complementarity property. We demonstrate that this algorithm successfully solves problems with as many as 10,000 variables which cannot be tackled by other current algorithms.

[6] We propose a parallel implementation of the classical Lemke's algorithm for solving the linear complementarity problem. The algorithm is designed for a loosely coupled network of computers which is characterized by relatively high communication costs. We provide an accurate prediction of speedup based on a simple operation count. The algorithm produces speedups near p, where p is the number of processors, when tested on large problems as demonstrated by computational results on the CRYSTAL token-ring multicomputer and the Sequent Balance 21000 multiprocessor.

2. Technical Reports Written under Support of Grant AFOSR-86-0255

[1] O. L. Mangasarian & R. De Leone: "Parallel successive overrelaxation methods for symmetric linear complementarity problems and linear programs", Journal of Optimization Theory and Applications, Volume 54, 1987, 437-446.

[2] O. L. Mangasarian & R. De Leone: "Parallel gradient projection successive overrelaxation for symmetric linear complementarity problems and linear programs", UW Comp. Sci. Tech. Rept. 659, August 1986, to appear in Annals of Operations Research.

[3] R. De Leone & O. L. Mangasarian: "Serial and parallel solution of large scale linear programs by augmented Lagrangian successive overrelaxation", UW Comp. Sci. Tech. Rept. 701, July 1987, to appear in "Proceedings of Workshop of Advanced Computation Techniques, Parallel Processing and Optimization-Karlsruhe, February 23-25, 1987", Springer-Verlag 1988.

[4] Karen M. Thompson: "A parallel asynchronous successive overrelaxation algorithm for solving the linear complementarity problem", UW Comp. Sci. Tech. Rept. 705, July 1987.

[5] Karen M. Thompson: "A two-stage successive overrelaxation algorithm for solving the linear complementarity problem", UW Comp. Sci. Tech. Rept. 706, July 1987.

[6] Karen M. Thompson: "A parallel pivotal algorithm for solving the linear complementarity problem", UW Comp. Sci. Tech. Rept. 707, July 1987.

3. List of Equipment Purchased under Grant AFOSR-86-0255

Purchase Order#	Item(s)	Cost
040H23-5	Six Digital Equipment Corporation Vaxstation II RC @ $7,497.50	$44,985.00
	Six Ethernet Transceivers @ $162.50	975.00
	Six Transceiver Cables @ $67.50	405.00
	Transportation	456.81
040H48-2	Seven 8-Megabyte Ram Cards @ $1,400.00	9,800.00
	One 4-Megabyte Board @ $625.00	625.00
	Freight	27.00
496 H 145	Two THE Modems @ $132.45	264.90
	Freight & Mail	8.15
	Total	$57,546.86

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